Math 10A with Professor Stankova
Worksheet, Discussion #23; Wednesday, 10/18/2017
GSI name: Roy Zhao

Integration Problems

Example

1. Integrate $\int_0^1 x^{-1/2}\arctan(\sqrt{x})dx$.

Solution: First we u sub to make this problem easier. Let $u = \sqrt{x}$ so $du = \frac{1}{2\sqrt{x}}dx$ so $dx = 2\sqrt{x}du$. When $x = 0$, then $u = \sqrt{0} = 0$ and when $x = 1$, then $u = \sqrt{1} = 1$ so, we have that

$$\int_0^1 x^{-1/2}\arctan(\sqrt{x})dx = \int_0^1 2\arctan(u)du.$$

Now we can use integration by parts to get $r = \arctan(u)$ and $dt = 2du$ so $t = 2u$ and $dr = \frac{1}{1+u^2}$ so

$$\int_0^1 2\arctan(u)du = 2u\arctan(u)\mid_0^1 - \int_0^1 \frac{2u}{1+u^2}du.$$

Now we can v sub the second integral. Let $v = 1 + u^2$ so $dv = 2udu$. So, we have that the integral is equal to

$$2\cdot1\cdot\arctan(1) - 2\cdot0\cdot\arctan(0) - \int_{u=0}^{u=1}\frac{1}{v}dv = 2\arctan(1) - \ln|1+u^2|_0^1 = 2\arctan(1) - \ln 2 = \frac{\pi}{2} - \ln 2.$$

Problems

2. Integrate $\int_1^{e^\pi}\sin(\ln(x))dx$.

Solution: First u sub to get that $u = \ln(x)$ so $du = \frac{1}{x}dx$ and $dx = xdu = e^u du$. The new bounds become $\ln 1$ and $\ln e^\pi$ which are 0 and π. Thus, we have that the integral is equal to

$$\int_0^\pi e^u\cos(u)du.$$

Now we can integrate by parts with $r = \sin(u)$ and $dt = e^u du$ so $dr = \cos(u)du$ and $t = e^u$ to get

$$\int_0^{\pi} e^u \sin(u)du = e^u \sin(u)|_0^{\pi} - \int_0^{\pi} \cos(u)e^u du = -\int_0^{\pi} \cos(u)e^u du.$$

We can integrate by parts again to get that the integral is equal to

$$\int_0^{\pi} e^u \sin(u)du = -\cos(u)e^u \;|_0^{\pi} - \int_0^{\pi} \sin(u)e^u du.$$

So the integral is equal to

$$\int_0^{\pi} e^u \sin(u)du = \frac{e^{\pi} + e^0}{2} = \frac{e^{\pi} + 1}{2}.$$

3. Integrate $\int_0^{\pi/2} \sin(x)\cos(x)\sin(\sin(x))dx$.

Solution: First we u sub with $u = \sin(x)$ so $du = \cos(x)dx$ and this integral is equal to

$$\int_0^1 u\sin(u)du = -u\cos(u)\;|_0^1 + \int_0^1 \cos(u)du = -\cos(1) + \sin(1).$$

4. Integrate $\int_0^1 2x^3 \sin(x^2)dx$.

Solution: First u sub with $u = x^2$ to get

$$\int_0^1 2x^3 \sin(x^2)dx = \int_0^1 u\sin(u)du = -\cos(1)\sin(1).$$

The work is shown above.

5. Integrate $\displaystyle\int_0^1 \frac{\arcsin(\sqrt{x})}{2\sqrt{x}}dx$.

Solution: Let $u = \sqrt{x}$ so $du = 1/(2\sqrt{x})dx$ and we get that this integral is equal to

$$\int_0^1 \arcsin(u)du = u\arcsin(u)_0^1 - \int_0^1 \frac{u}{\sqrt{1-u^2}}du.$$

Let $v = 1 - u^2$ and $dv = -2udu$ so the integral is

$$1 \arcsin(1) - 0 \arcsin(0) + \int_1^0 \frac{1}{2\sqrt{v}} dv = \frac{\pi}{2} + \sqrt{v}|_1^0 = \frac{\pi}{2} - 1.$$

6. Integrate $\int_0^1 5x^4 \arcsin(x^5) dx$.

Solution: Let $u = x^5$ so $du = 5x^4 dx$ and so our integral is equal to

$$\int_0^1 \arcsin(u) du = \frac{\pi}{2} - 1,$$

from above.

7. Integrate $\int_{\tan(1)}^{\tan(e)} \frac{\ln(\arctan(x))}{1 + x^2} dx$.

Solution: u sub with $u = \arctan(x)$ so $du = \frac{dx}{1+x^2}$ and our integral becomes

$$\int_1^e \ln(u) du = u \ln(u)\ |_1^e - \int_1^e du = e \ln e - 1 \ln 1 - (e - 1) = e \ln e - e + 1.$$

8. Integrate $\int_{\pi/4}^{\arctan(e)} \sec^2(x) \ln(\tan(x)) dx$..

Solution: u sub with $u = \tan(x)$ and $du = \sec^2(x) dx$ to get

$$\int_1^e \ln(u) du = e \ln e - e + 1.$$